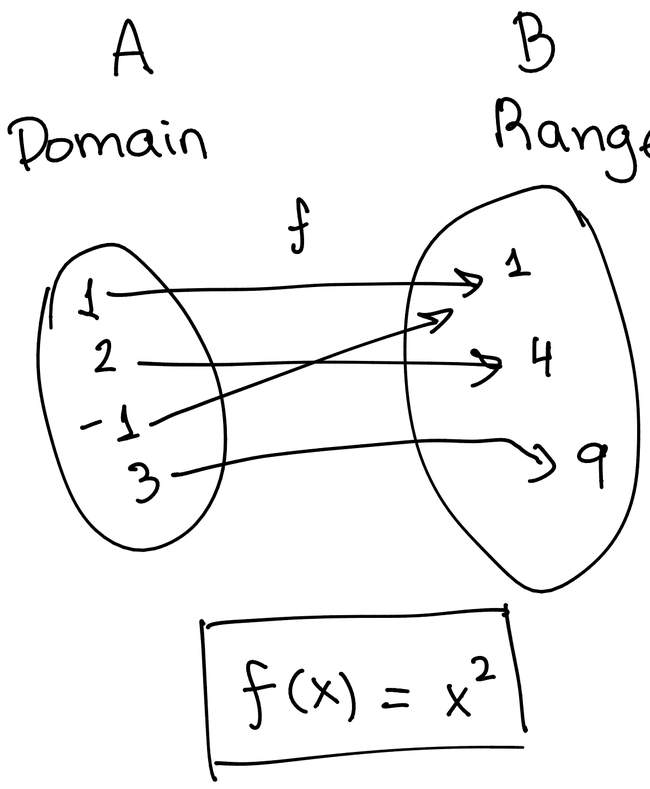
# Class 9 - § 3.2 Graphs of Functions

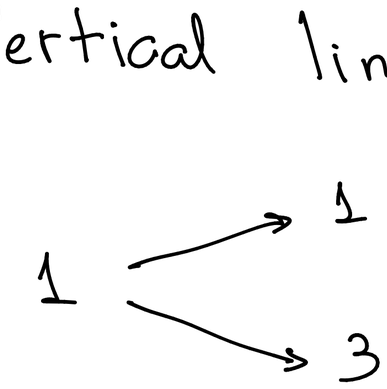
## Review

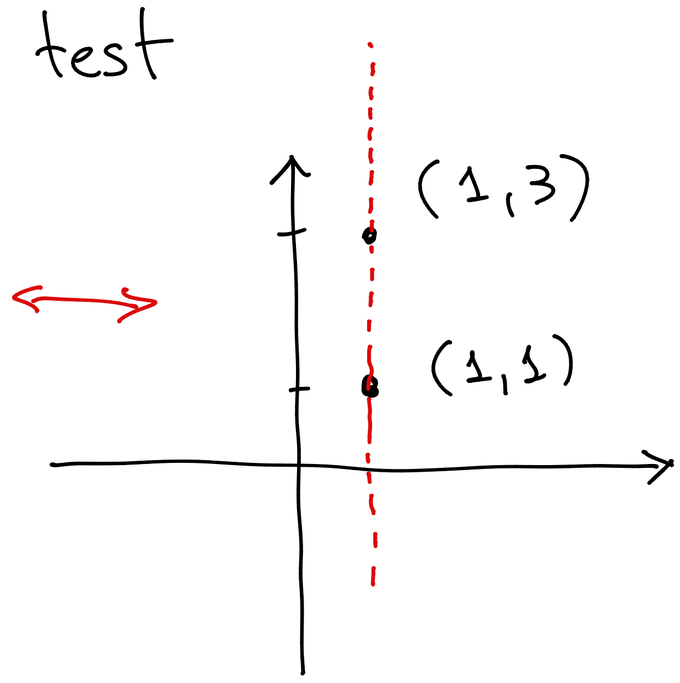
Relations & Functions ↪ every element in the domain has exactly ONE arrow.

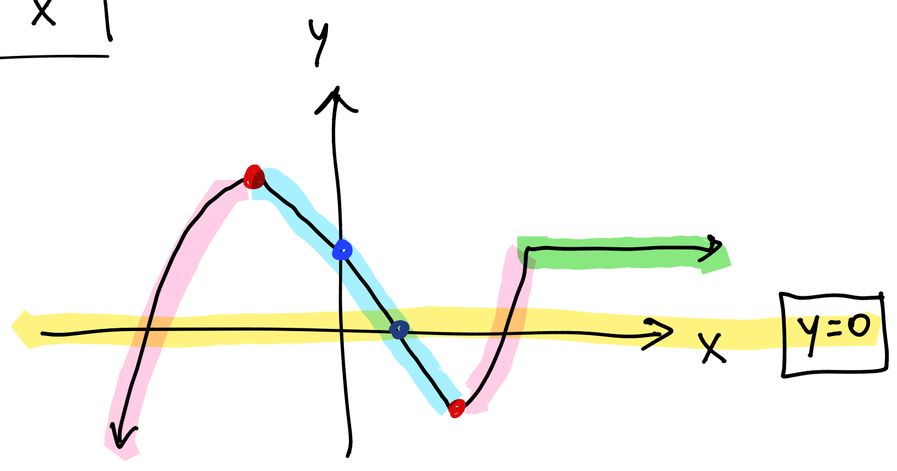



$f(x) = x^2$

Vertical line test




→ Factoring and solving higher order Polynomials

**Ex 1:** Factor $x^3 - x^2 - 2x + 2$.

$x^3 - x^2 - 2x + 2 = x^2(x-1) - 2(x-1) = (x-1)(x^2-2)$

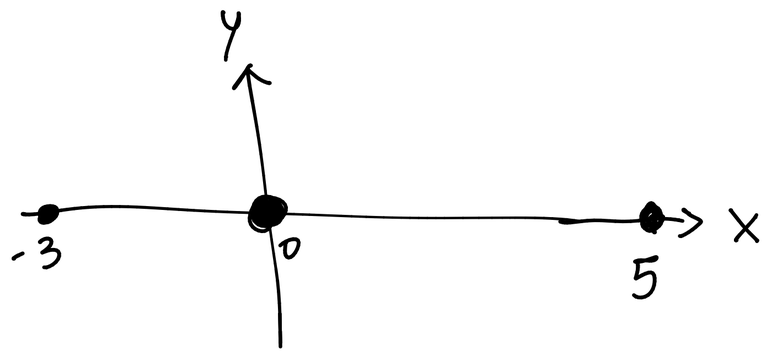
**Ex 2:** Factor $x^3 - 2x^2 - 15x$

$x^3 - 2x^2 - 15x = x(x^2 - 2x - 15) = x(x-5)(x+3) = 0$

$x=0$ $x=5$ $x=-3$

$y = f(x) = x^3 - 2x^2 - 15x$

$x = 0 \Rightarrow y = 0$



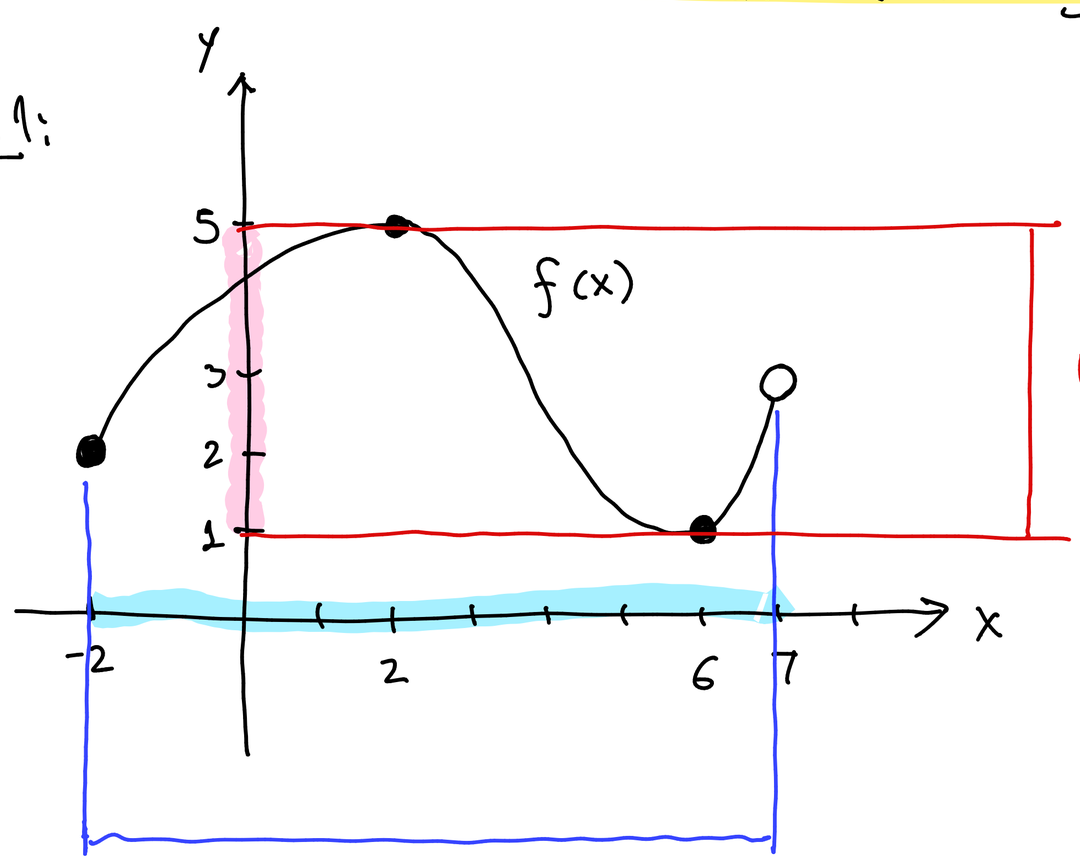
Regarding x-int. & y-int.:

| | x-int | y-int |
|---|---|---|
| $y = 5x^1 + 6$ → Linear (1 y & 1 x int.) | $-\frac{6}{5}$ | 6 |
| $y = x^2 + 2x + 1$ → Quadratic (1 y. & up to 2 x-int.) | $-1$ | 1 |
| $y = \sqrt{2x-1}$ → Root (at most 1 y & 1 x) | $\frac{1}{2}$ | DNE |
| $y = \frac{x+2}{3x-7}$ → Rational | $-2$ | $-\frac{2}{7}$ |

↳ x-int. ↔ numerator = 0

## Domain & Range

Ex 1:



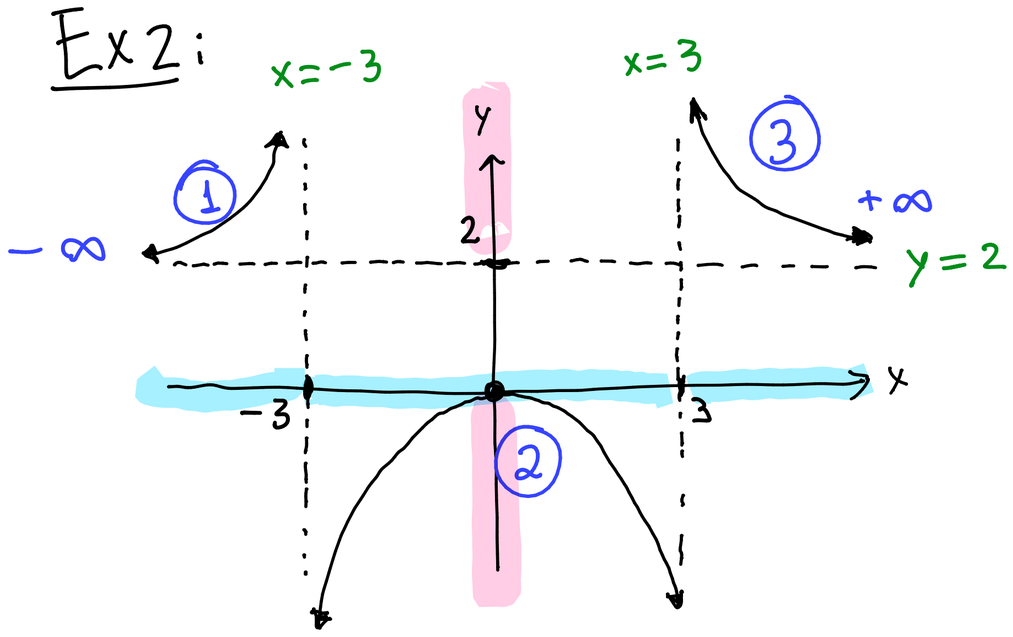

Range
[1,5]

Domain : all #'s (x-axis)

Range : all #'s (y-axis)

Domain
[-2, 7)

## Ex 2:




NOT use $x = \pm 3$

① $(-\infty, -3)$

② $(-3, 3)$

③ $(3, \infty)$

① & ③ $(2, \infty)$

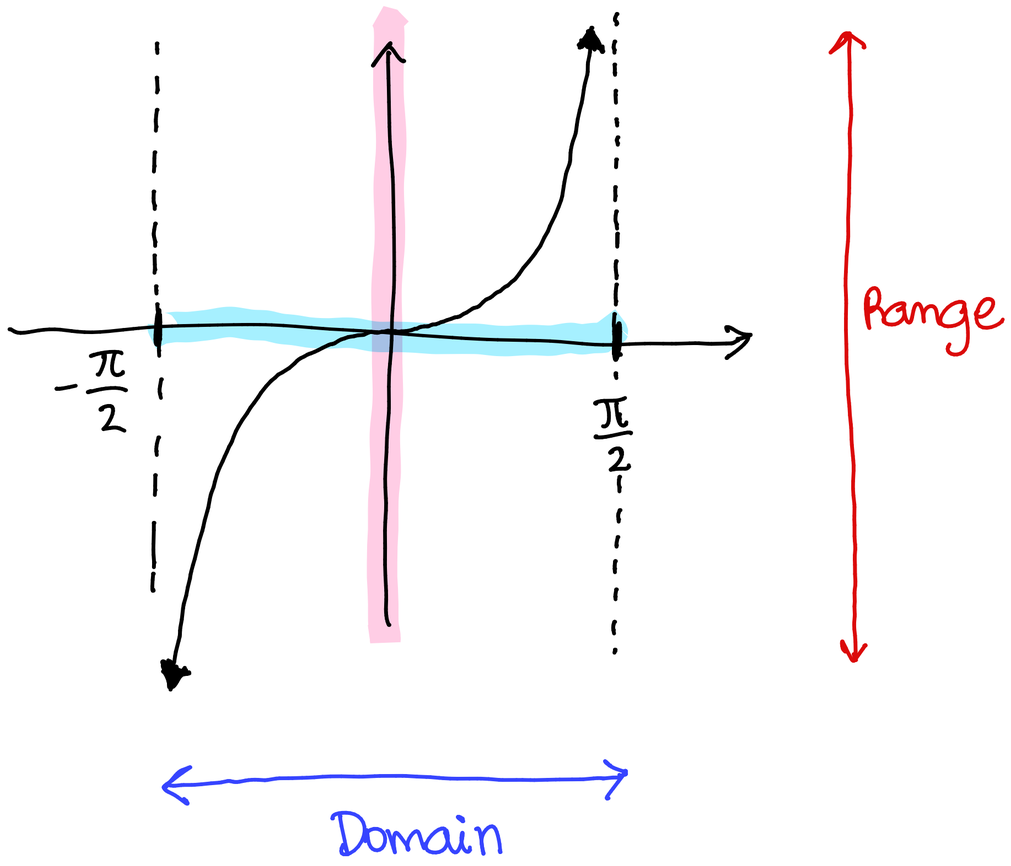
② $(-\infty, 0]$

Domain : $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

Range : $(-\infty, 0] \cup (2, \infty)$

## Ex 3:

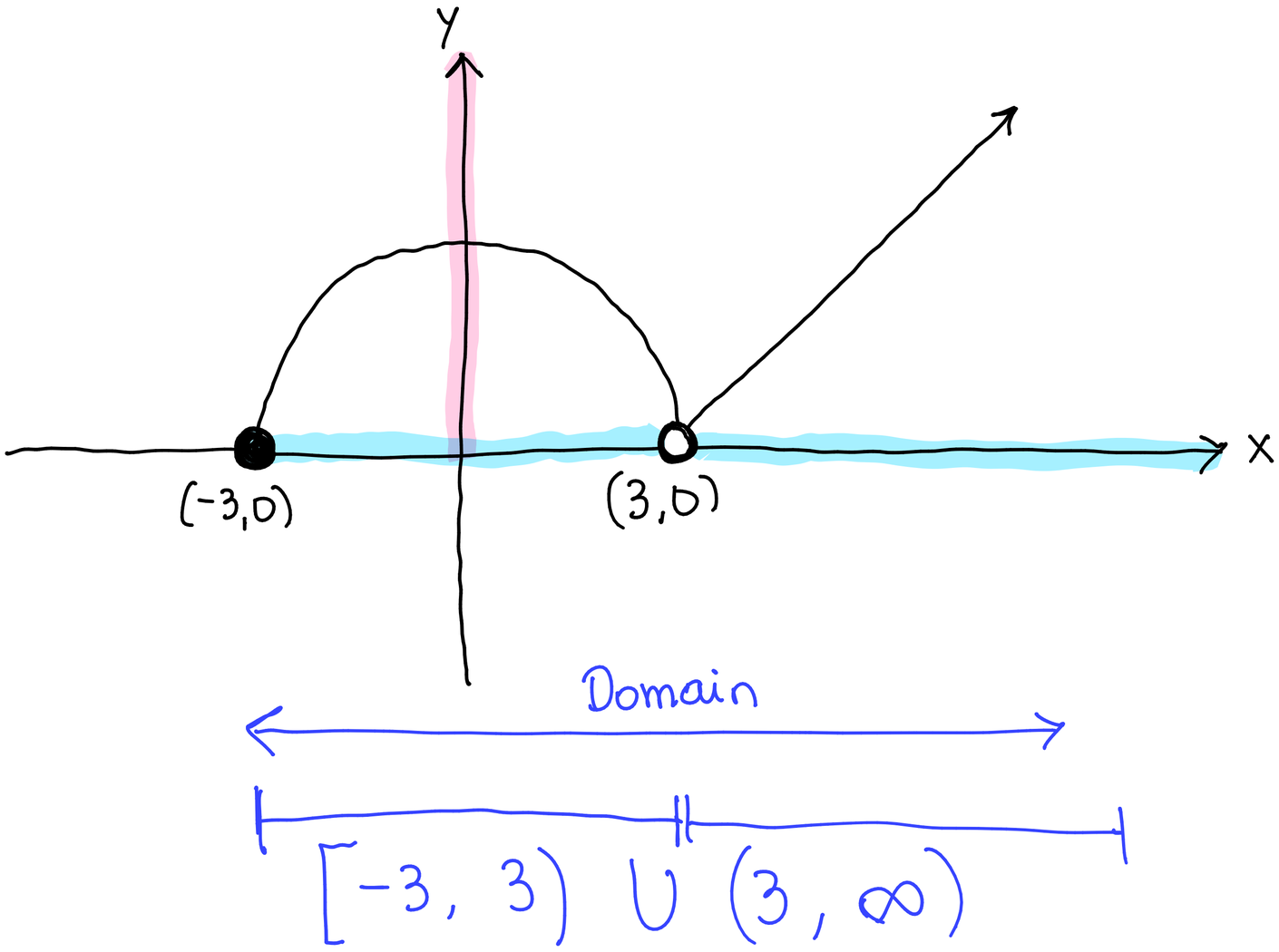
$f(x) = \tan(x)$




Domain: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

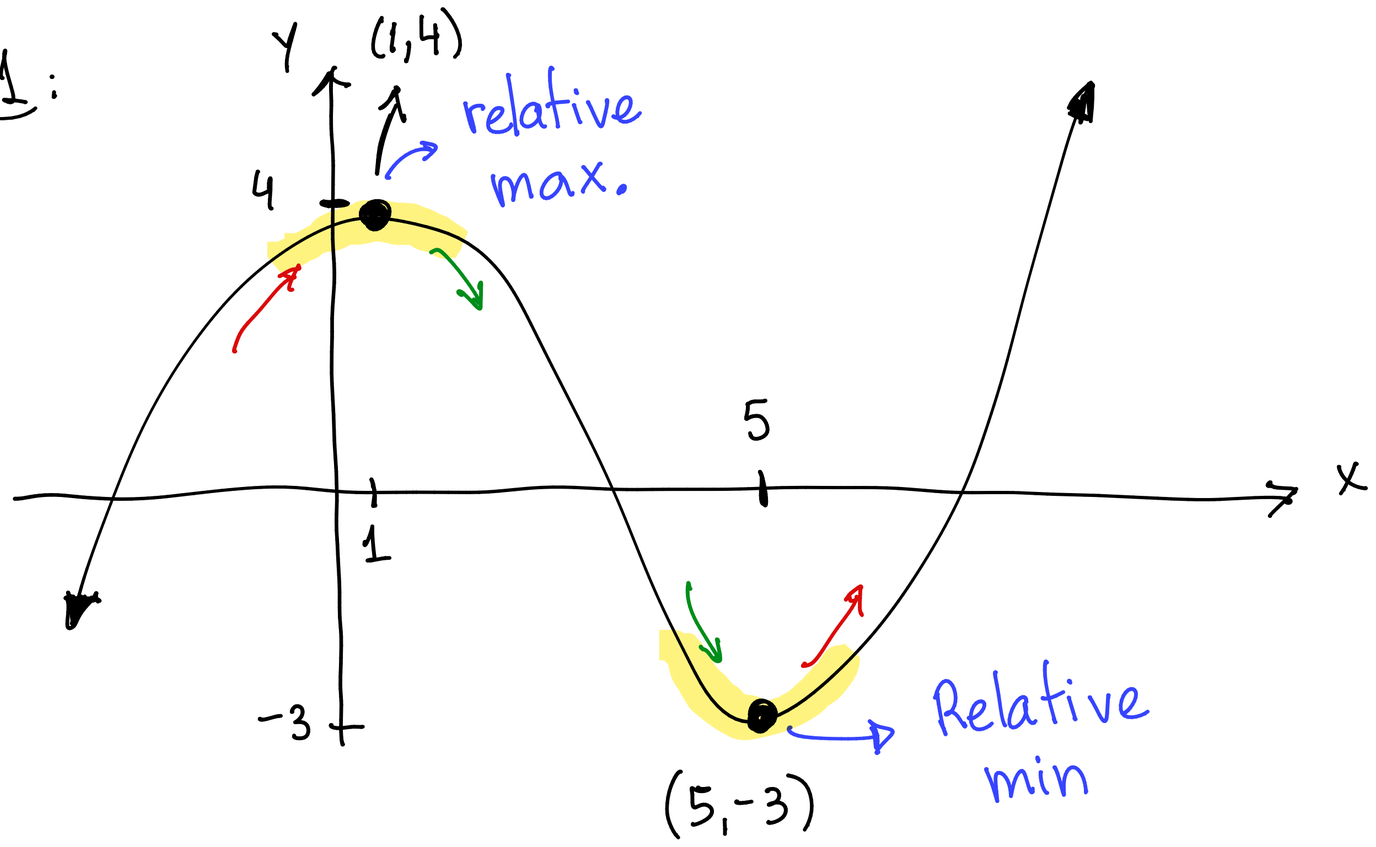
Range: $(-\infty, \infty)$

Ex 4:




→ Relative minimum & Relative maximum

Ex 1:




Relative max:

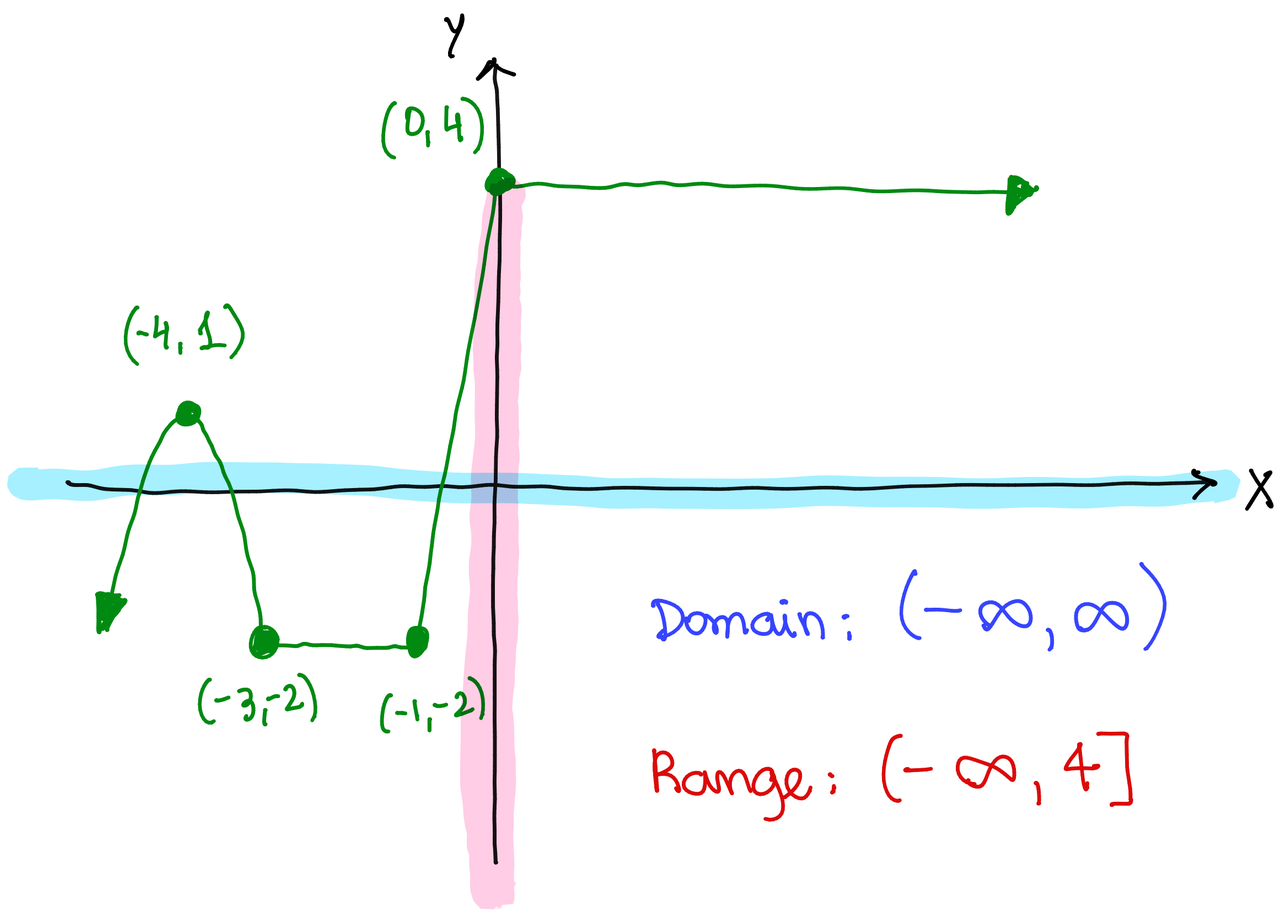
increasing to decreasing

Relative min:

decreasing to increasing

- the rel. max. happens at $x = 1$ & rel. max. value is 4
- the rel. min. happens at $x = 5$ & rel. min. value is $-3$
- Decreasing intervals: $(1, 5)$
- Increasing intervals: $(-\infty, 1)$ , $(5, \infty)$

## Ex 2: Find the following:




Domain: $(-\infty, \infty)$

Range: $(-\infty, 4]$

a) increasing intervals
$(-\infty, -4)$ , $(-1, 0)$

b) decreasing intervals
$(-4, -3)$

c) Constant intervals
$(-3, -1)$ , $(0, \infty)$

d) Relative Max.
$(-4, 1)$

e) Relative Min.
D.N.E.

→ Even & Odd functions

- Algebraically: Odd when $f(-x) = -f(x)$

  Even when $f(-x) = f(x)$

Ex: $f(x) = x^2 - 3$ & $g(x) = x^3 + x$

$f(-x) = (-x)^2 - 3$

$= (-1)^2 (x)^2 - 3$

$= x^2 - 3$

$= f(x)$ → even

all exponents even

$g(-x) = (-x)^3 + (-x)$

$= (-1)^3 (x)^3 - x$

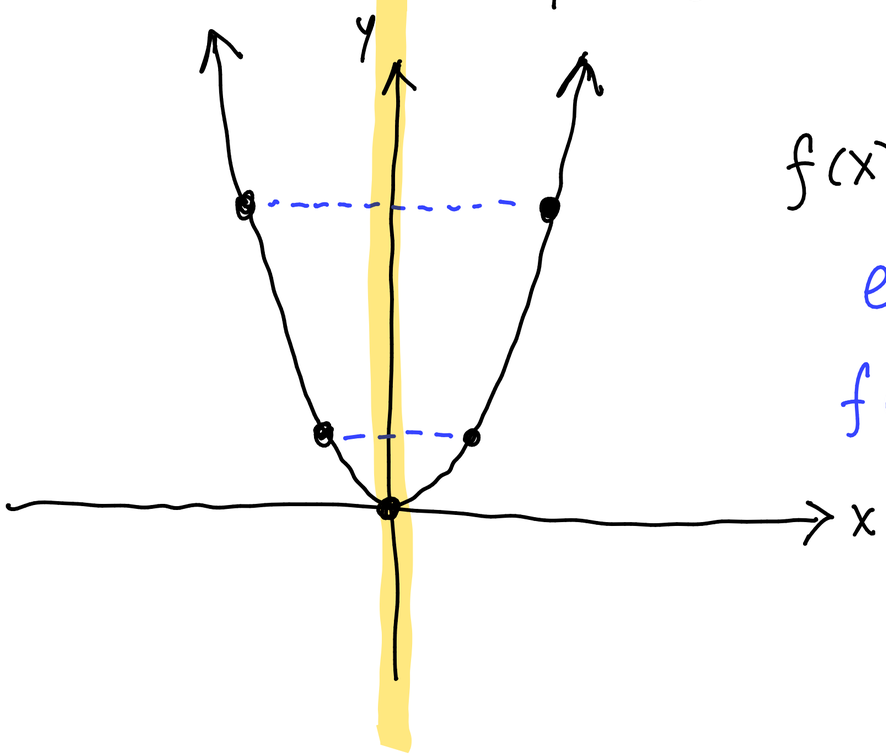
$= -x^3 - x$

$= -(x^3 + x)$

$= -g(x)$ → odd

all exponents odd

- Geometrically (graph):




Mirror

$f(x) = x^2$

even

$f(-x) = f(x)$

$f(0) = 0$

$f(-1) = 1$

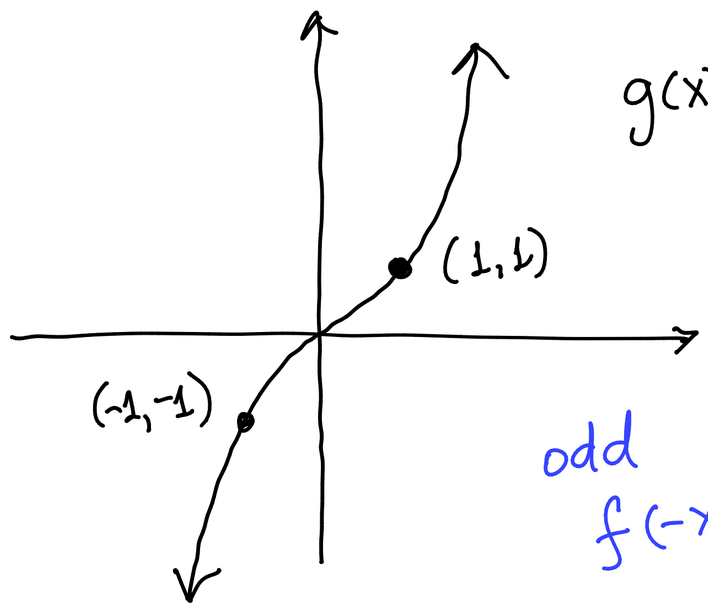
$f(+1) = 1$

$f(-2) = 4$

$f(2) = 4$

Mirror reflection around y-axis

180° rotation around the origin

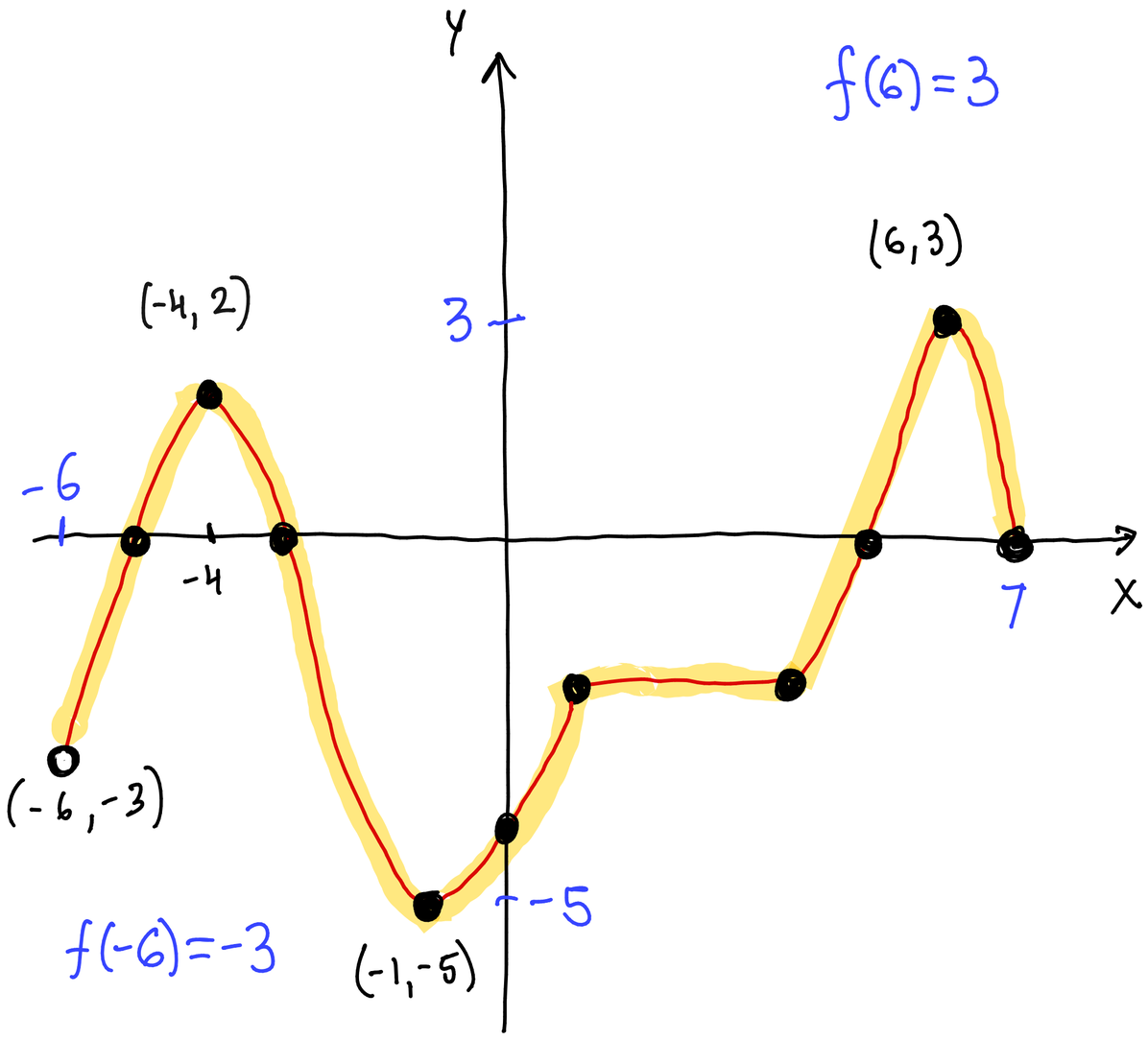



$g(x) = x^3$

odd

$f(-x) = -f(x)$

Exercise 1:




$f(6) = 3$

$f(-6) = -3$

a) Domain $(-6, 7]$

b) Range $[-5, 3]$

c) Increasing $(-6,-4)$, $(-1, 1)$, $(4, 6)$

d) Decreasing $(-4,-1)$, $(6, 7)$

e) Constant $(1, 4)$

f) Rel. Max. $(-4, 2)$ & $(6,3)$

g) Rel. Min. $(-1,-5)$

h) Symmetries not even, not odd

i) x-int. & y-int. $x = -5,-3, 5, 7$

$y = -4$